# Spectral theory for nonselfadjoint operators with applications to functional differential

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The aim of the lectures is to present the spectral properties of operators that appear naturally in the study of differential equations in Banach spaces. The operators either appear as the nonselfadjoint infinitesimal generator of an abstract Cauchy problem or as the monodromy operator of a periodic evolution equation. Solutions of the differential equation starting at an eigenvector or generalized eigenvector of the infinitesimal generator or monodromy operator give rise to special solutions of the differential equation. To study the qualitative properties of the solutions of these differential equations, it is important to know whether the operator associated with the differential equation has a complete span of eigenvectors and generalized eigenvectors. In the periodic case, completeness of the eigenvectors and generalized eigenvectors of the monodromy operator allows the possibility of a Floquet theory. Noncompleteness of the eigenvectors and generalized eigenvectors is closely related to the existence of solutions of the differential equation that decay faster than any exponential to zero, so-called small solutions. Several concrete applications of our main result are studied.

(1) Functional differential equations: large time behaviour.
In particular, for dynamical systems governed by feedback laws, time delays arise naturally in the feedback loop to represent effects due to communication, transmission, transportation or inertia effects. The introduction of time delays in a system of differential equations results in an infinite dimensional state space. The solution operator associated with a differential delay equation or functional differential equation is a nonself-adjoint operator defined on a Banach space and our main result can be applied to study the large time behaviour of linear autonomous en linear periodic functional differential equations.

(2) Parameter identifiability in functional differential equations.
Parameter identifiability is concerned with the question whether the parameters of a specific model can be identified from knowledge about certain solutions of the model, assuming perfect data. In this lecture we consider conditions for identifiability of parameters and time delays in infinite dimensional dynamical systems, for example, described by linear differential delay equations, assuming knowledge of particular solutions on bounded time intervals. The aim is to exploit methods from operator theory to formulate necessary and sufficient conditions which guarantee that the inverse problem has a unique solution.

(3) Feedback stabilization in functional differential equations.
Recently a lot of attention is devoted to dynamical systems with feedback, i.e., optical feedback lasers, phase-locked frequency synthesizers and wave equations with feedback stabilization at the boundary. In the implementation of a feedback system, it is very likely that time delays will occur in the feedback loop. It is therefore of vital importance to understand the sensitivity and robustness of the feedback system with respect to time delays in the closed loop. In joint work with Jack Hale, we have developed powerful tools, based on the radius of the essential spectrum of the semigroup associated with the open loop system, to study robustness properties of the closed loop system with respect to time delays in the loop.

(4) Functional differential equations of mixed type.

A Lattice Differential Equation (LDE) is a continuous-time infinite dimensional dynamical system with a discrete spatial structure modeled on a lattice. Such equations play an important role in modeling a variety of applications with spatial structure and can be found in chemical reaction theory, image processing and pattern recognition, material science and biology. Travelling waves in lattice differential equations naturally lead to linear systems of functional differential equations of mixed type with retarded and advanced arguments. As an initial value problem, these equations are not well-posed. Nevertheless, the basic linear theory including exponential dichotomies has recently been developed and will be presented in this lecture.